

MULTIVARIABLE SYNTHESIS WITH TRANSFER FUNCTIONS

Joseph L. Peczkowski
Bendix Energy Controls Division

ABSTRACT

A transfer function design theory for multivariable control synthesis is highlighted. The use of unique transfer function matrices and two simple, basic relationships - a synthesis equation and a design equation - are presented and illustrated. The basic idea of the method is straightforward, easy to understand and easy to apply.

This multivariable transfer function approach provides the designer with a capability to specify directly desired dynamic relationships between command variables and controlled or response variables. At the same time, insight and influence over response, simplifications and internal stability is afforded by the method. A general, comprehensive multivariable synthesis capability is indicated including nonminimum phase and unstable plants. Gas turbine engine examples are used to illustrate the ideas and method.

INTRODUCTION

The concept of transfer function has been a mainstay of control engineering and design. A primary motivation to use transfer functions for multivariable design is to increase insight, choices and simplifications so that a designer may interact with multivariable systems in a more direct and integrated manner. The multivariable transfer function idea was applied to jet engine control by Boksenbom and Hood (1) and Feder and Hood (2) about mid-century. Practical computation with transfer function matrices was a difficult issue at that time; and, the question of how to extend performance specifications to matrices of transfer functions posed another difficulty which is still not completely resolved. In this paper, the use of transfer functions for design of controller dynamics for linear multivariable models of turbine engines is reexamined.

Control system design methods in the frequency domain traditionally have been of two types. In the first type, the designer works indirectly to adjust open loop characteristics so that, when the loop is closed, an acceptable system results. In the second type, the designer works directly from the closed loop specifications

to the specific controller dynamics required. We refer here to this second type method as a synthesis method. A classical discussion of both methodologies may be found in Truxal (3).

Two multivariable transfer function system relationships are derived- a design equation and a synthesis equation. The synthesis equation is used to display internally stable closed loop response possibilities; the design equation is used to compute and simplify explicit controller dynamics. Gas turbine engine examples illustrate ideas and methodology.

MULTIVARIABLE CONTROL SYNTHESIS

The basic notion of multivariable control synthesis with transfer functions is straightforward and easy to understand. Consider Figure 1, a block diagram for a unity negative feedback multivariable structure with no disturbances. References, error, plant input and plant output are designated r , e , u and y respectively. Assume the plant has equal numbers of inputs and outputs, thus $P(s)$ is a square matrix of transfer functions. This assumption is not nearly as restrictive as one might suppose at the outset. More on this later. The controller $G(s)$ is also square.

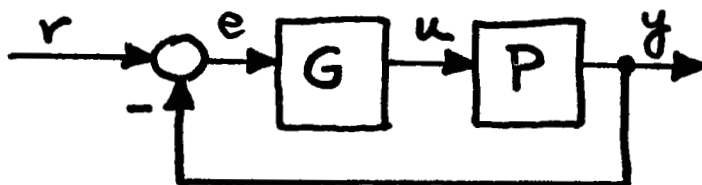


Figure 1. Unity Feedback Structure

The problem is, given plant $P(s)$, to design a controller $G(s)$ to achieve desired, internally stable, closed loop response $T(s)$ as indicated in Figure 2. The objective is to design $G(s)$ so that closed loop response $T(s)$ is achieved in such a way that designer choices, insight and influence are made available and remain accessible. References (4) and (5) can provide more details for the interested reader.

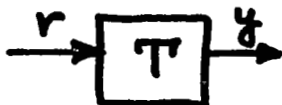


Figure 2. Desired Response

A Design Equation

From Figure 1, the total response of the unity feedback loop is

$$y = P G (I + P G)^{-1} r \quad (1)$$

The desired response is

$$y = T r \quad (2)$$

Combining equations (1) and (2) and solving for $G(s)$ gives the controller

$$G = P^{-1} T (I - T)^{-1} \quad (3)$$

This equation may be written in a convenient, compact form

$$G = P^{-1} Q \quad (4)$$

where Q , a performance matrix, is defined by $Q = T (I - T)^{-1}$. Equation (4) is named the design equation for controllers $G(s)$ under the unity feedback structure of Figure 1. The design equation simply and clearly indicates that controller design focuses upon the properties of the plant inverse, $P(s)^{-1}$, and how they interact with $Q(s)$, the performance matrix.

For a unity feedback structure as in Figure 1, and the case of decoupled response forms where the response matrix T is diagonal, a Q - T transform table conveniently exhibits elements of the performance matrix, q_{ii} , corresponding to given elements of the response matrix, t_{ii} . Table I lists some standard response forms and related performance element forms for unity feedback loop structures.

TABLE 1

A Q vs. T Transform Table - Unity Feedback, Decoupled Response

DESIRED $t_{ii}(s)$	$q_{ii}(s)$
$\frac{1}{Ts + 1}$	$\frac{1}{Ts}$
$\frac{1}{(T_1 s + 1)(T_2 s + 1)}$	$\frac{1/(T_1 + T_2)}{(\frac{T_1 T_2}{T_1 + T_2})s + 1}$
$\frac{W_n^2}{s^2 + 2\zeta W_n s + W_n^2}$	$\frac{K}{s(Ts + 1)}$; $K = W_n/2\zeta$, $T = 1/s\zeta W_n$
$\frac{1}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)}$ $T_1 \quad T_2 \quad T_3$	$\frac{1/(T_1 + T_2 + T_3)}{s((\frac{T_1 T_2}{T_1 T_2 T_3})s + 1)(T_3 s + 1)}$

More General Feedback Structure

To effect control of a plant it is necessary to use actuators to drive inputs and to use sensors to measure outputs. Moreover, sensors and actuators can introduce significant dynamical effects into signal paths of the loop. Therefore, a more general feedback structure, which accommodates these effects, is shown in Figure 3.

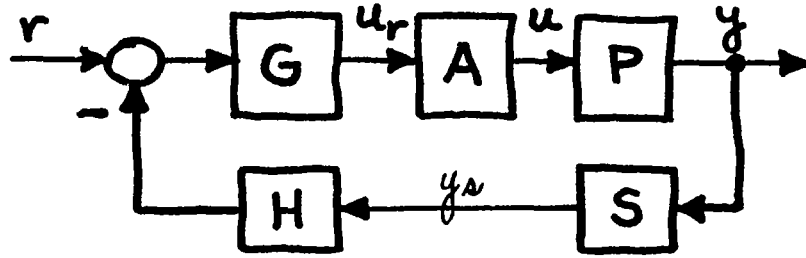


Figure 3 General Feedback Loop

$A(s)$ and $S(s)$ are diagonal actuator and sensor matrices, respectively. The output y is sensed and becomes y_s ; the input request, u_r , commands the actuators to produce the plant input, u .

From Figure 3, overall response of the loop is

$$y = (I + PAGHS)^{-1} PAG r \quad (5)$$

The desired response is

$$y = T r \quad (6)$$

Combining equations (5) and (6) and solving for the controller G

$$G = A^{-1} P^{-1} T (I - HST)^{-1} \quad (7)$$

A performance matrix

$$Q = T (I - HST)^{-1} \quad (8)$$

is easily identified. The design equation becomes

$$G = A^{-1} P^{-1} Q \quad (9)$$

This is a design equation for feedback systems depicted in Figure 3 with the performance matrix Q per equation (8). Equation (9) is a generalization of equation (4); controller dynamics are determined by the characteristics of the actuated plant inverse, $(PA)^{-1}$, and the performance matrix, Q . The plant inverse transfer function matrix is a key element in the design equation. What about the existence of the plant inverse? Is this a serious restriction to transfer function design?

The Plant Inverse

Fortunately, the need for existence of the plant inverse turns out to be not a significant limitation of the design equation. Rather, we can indicate to the contrary that the plant inverse establishes and displays vital plant characteristics needed to effect successful closed loop control design. Four system and plant features, essential for design, are established and identified by the plant inverse transfer function:

1. meaningful multivariable control (6)
2. plant trackability (8)
3. multivariable plant zeros (9)
4. cancellations and simplifications

Rekasius (6) and Wonham and Morse (7) have shown that if the number of plant inputs equals the numbers of its outputs and if $P(s)$ is full rank, i.e., $P(s)^{-1}$ exists, then one has both a meaningful multivariable control problem and necessary and sufficient conditions for existence of a physically realizable controller that decouples the system. If the number of plant outputs is greater than the number of its inputs, some of the outputs cannot be controlled independently. Even if the number of plant inputs exceeds the number of its outputs, independent control of all outputs is not possible if the plant transfer function matrix, $P(s)$ is not full rank (6).

R. J. Leake, et al (8) define a step trackable linear multivariable plant as one which can asymptotically achieve any constant steady-state output with a bounded control. It is shown that step trackability for proper rational continuous square plants is equivalent to the conditions that:

1. the plant is invertible
2. the plant has no multivariable zeros at the origin ($s = 0$)

Leake goes on to show the significance of step trackability by demonstrating that internally stable, decoupled closed loop design is possible if and only if the plant is step trackable.

Importantly, the multivariable zeros of a plant, $P(s)$, are the poles of the inverse, $P(s)^{-1}$, Wyman and Sain (9). Therefore, multivariable plant zeros are readily identified from the factored form of the inverse transfer function matrix.

Thus, existence of the plant inverse assures conditions needed to effect design. Moreover, the plant inverse matrix provides essential design information about plant trackability, about plant multivariable zeros and about existence of meaningful and internally stable closed loop control realizations.

A Synthesis Equation

Little has been said about loop stability. From another view of the problem of synthesis of closed loop controllers, a synthesis equation will be derived which can be used to establish existence of internally stable closed loop controllers. The synthesis equation was first proposed by Dr. R. J. Leake of Fresno State. Connection of the equation to current system theory (10) and internal stability was made by Dr. M. K. Sain of Notre Dame. The author is happy to acknowledge continuing collaborations and discussions with Professors Sain and Leake on multivariable synthesis with transfer functions.

Consider Figure 4 where r denotes request, u denotes control action, and y denotes response. Under broad assumptions, there exist linear operators $T: R \rightarrow Y$

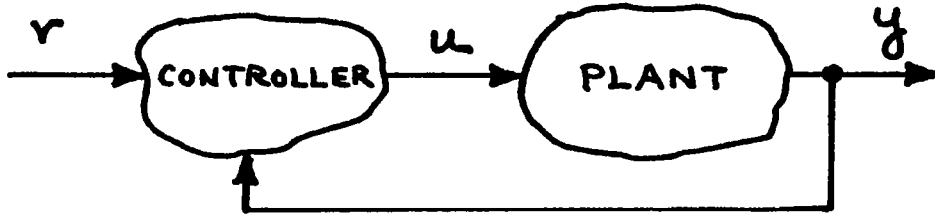


Figure 4 A General Control System

and $M: R \rightarrow U$, where R , U , and Y may be understood as $R(s)$ -vector spaces of finite dimension such that (5)

$$y = T r, u = M r \quad (10)$$

The plant can be understood in terms of an operator $P: U \rightarrow Y$, such that

$$y = P u \quad (11)$$

Combining equations (10) and (11) obtains the relationship

$$T = P M \quad (12)$$

Bengtsson(10) proves that internally stable feedback realizations of systems depicted by Figure 4 exist if and only if M is proper and stable and T is proper and stable.

Imposing a tracking requirement, as for example that y should asymptotically track step responses, then P must be epic. If in addition the number of inputs equals the number of outputs, then P is monic also. Therefore, we can address an inverse total synthesis problem (ITSP) (5) which is governed by the synthesis equation

$$M = P^{-1} T \quad (13)$$

Note that equation (13) is similar in form to design equation (4) or (9).

Two Basic Equations

Two equations form a basis for multivariable synthesis with transfer functions:

- the synthesis equation $M = P^{-1} T$
- the design equation $G = P^{-1} Q$

The idea is, for given plant P , to select proper and stable T so that M is also proper and stable. This insures existence of internally stable controllers. Thus, the synthesis equation displays all possible responses T which have internally stable feedback realizations.

Feedback realizations of M , by controller dynamics G and H , as indicated in Figures 1 and 3, are obtained by applying the controller dynamics design equation $G = P^{-1} Q$. The response matrix T maps to the performance matrix $Q = PG$ in Figure 1 and $Q = PAG$ in Figure 3. The issue of internal stability of closed loop realizations is still under study (4),(5) . However, applications of the synthesis and design equations to numerous examples, including nonminimum phase plants, suggest the conjecture that if M and T are proper and stable, and if no cancellations of right hand plane poles and zeros occur in the open loop matrix products $PAGHS$ (Figure 3), then internal stability of the closed loop is assured. In any event, in the absence of a complete general theory and proof, doubts on internal stability can be resolved in practise by computer simulations of specific closed loop realizations.

The foregoing ideas and use of the synthesis and design equations are illustrated by examples.

DESIGN EXAMPLES

Two turbine engine examples are given to demonstrate linear multivariable

synthesis with transfer functions. The first design example uses a third order research model of General Electric's J-85 engine with two inputs and two outputs. The second example uses a sixth order model of Pratt & Whitney's F100 engine (11) with four inputs and four outputs. It is a pleasure to acknowledge the computational support of E. J. Olbey, S. A. Stopher and J. W. Wildrick of the Bendix Energy Controls Division.

Example 1

A transfer function matrix of the J-85 engine at sea level, 100% speed condition is given by

$$y = P(s) u$$

where y is the output vector and u the input vector. The output vector $y'=(N,T)$ where N is rotor speed, RPM, and T is turbine temperature, °F. The input vector $u' = (W_f, A_j)$ where W_f is fuel flow, pounds per hour, PPH, and A_j is nozzle area, in². The plant transfer function is

$$P(s) = \begin{bmatrix} \frac{5.6}{(.61s+1) (.016s+1)} & \frac{55.7}{(.61s+1) (.06s+1)} \\ \frac{.17 (1.3s+1)}{(.61s+1)(.23s+1) (.016s+1)} & \frac{-1.9}{(.61s+1)(.23s+1)(.06s+1)} \end{bmatrix}$$

Response Specification

A closed loop controller is desired to control the engine so that system response to a step: 1. settles in one second, 2. has no overshoot, 3. obtains zero steady state error. Also, decoupling of the output is desired thus the response matrix T is diagonal. The control problem is pictured by the diagram in Figure 5.

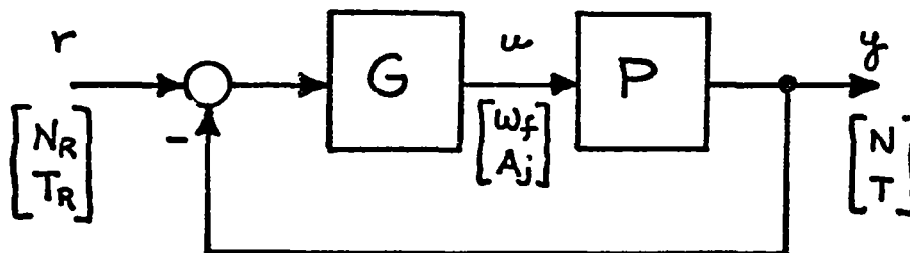


Figure 5 J-85 Engine and Control

Plant Inverse

The plant inverse matrix is calculated first. The plant inverse exists, therefore, a

$$P(s)^{-1} = \begin{bmatrix} .094 (.016s+1) & 2.8 (.016s+1) (.23s+1) \\ .0084 (.06s+1)(1.3s+1) & -.28(.06s+1)(.23s+1) \end{bmatrix}$$

meaningful problem is posed; and, it is possible to shape the response of the outputs independently with the available inputs. Decoupled response is possible. The plant has no multivariable zeroes since P^{-1} has no poles. Thus P and P^{-1} indicate that at the given condition, the J-85 engine is a stable plant with no multivariable zeros. No possibility for rhp cancellations from the plant.

Synthesis Equation

The synthesis equation, $M = P^{-1}T$, is applied to determine possible loop responses. Internally stable realizations exist if and only if both M and T are proper and stable. For diagonal T

$$M = \begin{bmatrix} 0.94 (.016s+1) & t_{11} & 2.8(.016s+1)(.23s+1) & t_{22} \\ .0084(.06s+1)(1.3s+1) & t_{11} & -.28(.06s+1)(.23s+1) & t_{22} \end{bmatrix}$$

M is proper and stable when $t_{11}, t_{22} = \frac{K (*)}{(T_1s+1) (T_2s+1) (*)}$ form.

Selection of Response Matrix

To meet response specifications, the response matrix T is selected and structured initially as follows:

- diagonal - decoupled response
- predominant time constant = .25 sec - one second settling
- gain = 1 - zero steady state error
- $t_{ii} = 1 / (.25s \pm 1) (*)$ - to satisfy synthesis equation

The above structure of T implies that the performance matrix $Q = T(I-T)^{-1}$ is also diagonal and $q_{ii} = K/s (*)$. Of course t_{11} and t_{22} can be chosen differently and independently.

Design Equation

Controller dynamics are computed by the design equation $G = P^{-1}Q$. For diagonal Q

$$G = \begin{bmatrix} .094 (.016s+1) & q_{11} & 2.8 (.016s+1)(.23s+1) & q_{22} \\ .0084(.06s+1)(1.3s+1) & q_{11} & -.28(.06s+1)(.23s+1) & q_{22} \end{bmatrix}$$

The above matrix form clearly shows that G is simplified if $q_{11} = q_{22} = K/s(.06s+1)(.016s+1)$. Combining the q_{ii} and t_{ii} requirements gives

$$t_{11} = t_{22} = \frac{1}{(.25s+1)(.08s+1)(.016s+1)}$$

and

$$q_{11} = q_{22} = \frac{2.89}{s(.06s+1)(.016s+1)}$$

Then controller dynamics are

$$G(s) = \begin{bmatrix} \frac{.27}{s(.06s+1)} & \frac{8.1(.23s+1)}{s(.06s+1)} \\ \frac{.02(1.3s+1)}{s(.016s+1)} & \frac{-.81(.23s+1)}{s(.016s+1)} \end{bmatrix}$$

Verification

Computer simulation of the J-85 closed loop system (Figure 5) with the above controller dynamics verifies the desired response $T = 1/[(.25s+1)(.08s+1)(.016s+1)]$. Figure 6 shows the response of the J-85 engine-control system to a 500 RPM step in speed request N_R only. Response specifications and decoupling are achieved. Figure 7 shows system response for 500 RPM step in speed request N_R and -50 degree step in temperature request T_R .

Example 2. F100 Turbofan Engine

An extensive set of linear state descriptions of the F100 turbofan engine were given by Miller and Hackney (11). In this example a reduced model at sea level, 67 degree power lever condition is controlled. This example illustrates a realistic design situation including engine, actuators and sensors. Use of approximate cancellations to simplify the controller is also illustrated.

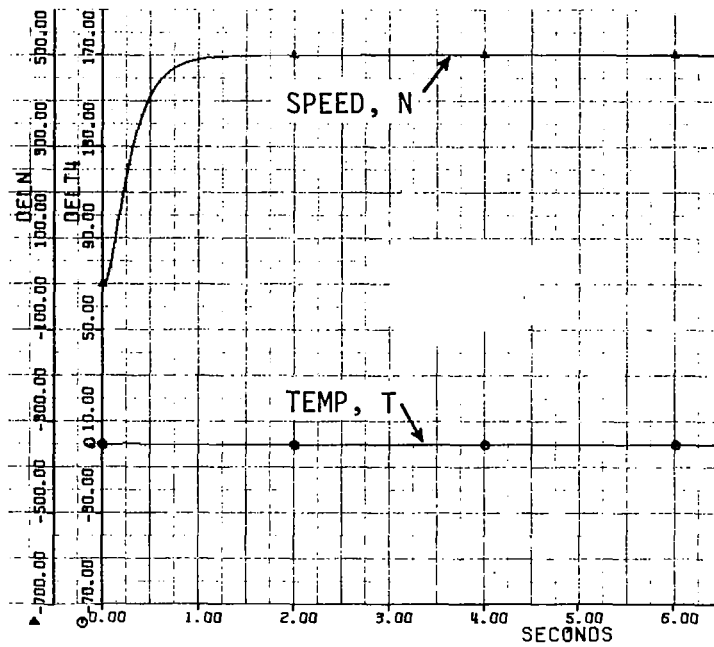


Figure 6. J-85 Response to 500 RPM Step

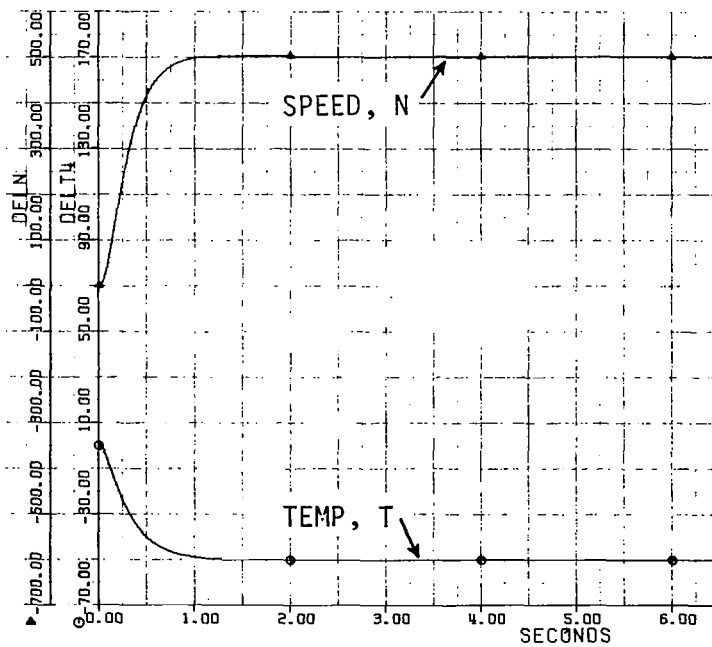


Figure 7. J-85 Response to 500 RPM Step and -50° Temp. Step

Engine Dynamics

A reduced state model of the F100 engine at sea level, 67 degree power lever condition is

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}$$

$$\mathbf{y} = \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{u}$$

where

$$\mathbf{A} = \begin{bmatrix} -4.064 & 3.895 & -470.5 & 7.971 & 5.294 & -3.005 \\ .03718 & -2.958 & -59.13 & .1727 & 2.08 & 12.48 \\ .03389 & .0067 & -4.442 & .0059 & .1474 & .0985 \\ 1.164 & -2.646 & -331.6 & -50.05 & -.473 & -11.36 \\ .05174 & -.1176 & -14.74 & -2.001 & -2.021 & -.505 \\ .00184 & .0036 & -.601 & .00008 & .0009 & -.666 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} .8686 & -14.51 & -96.14 & 9.246 \\ .9096 & -58.46 & -1.053 & -60.15 \\ -.007994 & -79.66 & 1.2 & .3673 \\ 5.643 & -112.2 & -18.23 & 41.53 \\ .2508 & -4.99 & -.8106 & 1.846 \\ .01 & -.3166 & -.02915 & .07426 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & 0 \end{bmatrix}$$

The states, \mathbf{x} , inputs, \mathbf{u} , and outputs, \mathbf{y} , are

$x_1 = N_1$, fan speed, RPM

$x_2 = N_2$, compressor speed, RPM

$x_3 = P_7$, augmentor pressure, PSI

$x_4 = T_{h1}$, fan turbine temperature (fast), °F

$x_5 = T_{1c}$, fan turbine temperature (slow), °F

$x_6 = T$, burner temperature (slow), °F

$u_1 = WF$, fuel flow, PPH

$u_2 = AJ$, exhaust nozzle area, FT^2

$u_3 = CIVV$, inlet vane position, DEG

$u_4 = RCVV$, compressor vane position, DEG

$y_1 = N_1$, fan speed, RPM

$y_2 = N_2$, compressor speed, RPM

$y_3 = P_7$, augmentor pressure, PSI

$y_4 = FTIT$, fan turbine inlet temperature, °F

The transfer function matrix of the engine ($y = Pu$) is $P = C(sI - A)^{-1}B + D$ and is shown in Figure 8. In the figure the notation $(Ts + 1) = (T)$ and $(As^2 + bs + 1) = (a, b)$ is used to save space. A stable, sixth order plant is indicated. The reader will note that the poles corresponding to the time constants (1.43) and $(.491)$ may be eliminated by approximate cancellations; thus, the essential dynamics are fourth order.

$$P = \begin{bmatrix} .408(1.45)(.501)(.301)(.132)(.009) & 1220(1.43)(.494)(.302)(.0175)(-.0004) & -30.6(1.43)(.493)(.302)(.034)(.019) & -10.8(1.41)(.466)(.300)(-.039)(.014) \\ .326(1.24)(.488)(.03)(.269)(.019) & 198(1.19)(.486)(.253)(.019)(-.013) & -1.98(.027)(.290, 1.08)(.021)(.012) & -19.1(1.50)(.493)(.027)(.732)(.020) \\ .0025(1.32)(.483)(.224)(-.131)(.021) & -7.80(1.47)(.501)(.341)(.256)(.02) & .0248(1.95)(.999)(.475)(.329)(.02) & -.0160(1.63)(-1.44)(.51)(.122)(.02) \\ .095(1.56)(.470)(.377)(.028)(.254) & 72.6(1.56)(.470)(.329)(.196)(-.004) & -1.23(1.55)(.470)(.088)(.666)(.031) & 1.83(1.54)(.470)(.118)(.035)(.144) \end{bmatrix}$$

(1.43)(.491)(.302)(.02)(.0297)(.236)

Figure 8. F100 Transfer Function Matrix

The plant inverse matrix is shown in Figure 9. The factored form indicates that the inverse has two poles associated with time constants (1.55) and $(.470)$ which are zeros of the plant. Again, these factors approximately cancel from the plant inverse matrix.

$$P^{-1} = \begin{bmatrix} -.334(1.55)(.470)(.109) & .897(1.55)(.470)(.119) & 42.6(1.55)(.470)(-.007) & 7.75(1.54)(.500)(.019) \\ -.000245(1.54)(.655)(.470) & .000505(1.54)(.470)(-.172) & -.129(1.55)(.470)(.097) & .00270(1.41)(.462)(.008) \\ -.046(1.55)(.470)(.235) & .0450(1.55)(.470)(-.010) & -4.50(1.55)(.470)(.0003) & .160(1.56)(.489)(.009) \\ -.00350(1.50)(.470)(.061) & -.0365(1.57)(.470)(.403) & -.138(1.50)(.470)(-.056) & .144(1.27)(.491)(.019) \end{bmatrix}$$

(1.55) (.470)

Figure 9. F100 Inverse Matrix

Response Specification

Assume the output response specifications of the F100 engine are

1. decoupled system, 2. step response settles in 1 second, 3. no overshoot, 4. zero steady state error. The desired feedback structure is shown in Figure 10.

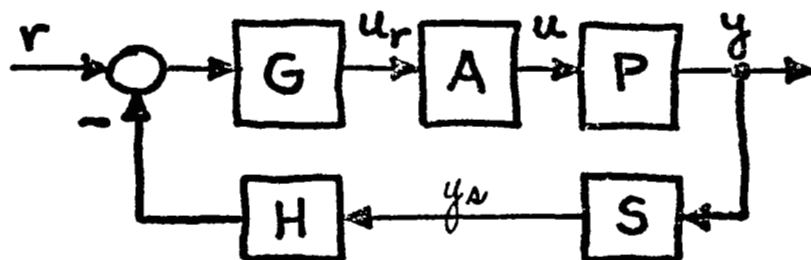


Figure 10. General Feedback Structure

Actuator dynamics are given by $u = A u_r$ and sensor dynamics are given by $y_s = S y$ where

$$A = \begin{bmatrix} \frac{1}{.05s+1} & 0 & 0 & 0 \\ 0 & \frac{1}{.2s+1} & 0 & 0 \\ 0 & 0 & \frac{1}{.1s+1} & 0 \\ 0 & 0 & 0 & \frac{1}{.1s+1} \end{bmatrix}; \quad S = \begin{bmatrix} \frac{1}{.02s+1} & 0 & 0 & 0 \\ 0 & \frac{1}{.02s+1} & 0 & 0 \\ 0 & 0 & \frac{1}{.02s+1} & 0 \\ 0 & 0 & 0 & \frac{1}{.5s+1} \end{bmatrix}$$

Synthesis Equation

The synthesis equation $M = A^{-1}P^{-1}$ becomes

$$M = \begin{bmatrix} -.33(.11)(.05) & t_{11} & .90(.12)(.05) & t_{22} & 42(-.01)(.05) & t_{33} & 7.8(.02)(.05) & t_{44} \\ -.0002(.66)(.2) & t_{11} & .0005(-.17)(.2) & t_{22} & -.13(.1)(.2) & t_{33} & .003(.01)(.2) & t_{44} \\ -.046(.24)(.1) & t_{11} & .045(-.01)(.1) & t_{22} & -4.5(.1) & t_{33} & .16(.01)(.1) & t_{44} \\ -.0035(.061)(.1) & t_{11} & -.037(.40)(.1) & t_{22} & -.14(-.06)(.1) & t_{33} & .14(.02)(.1) & t_{44} \end{bmatrix}$$

M is stable and proper when the t_{ii} form is $K/(Ts+1)(Ts+1)$. Based on the response specifications, $t_{ii} = 1/((.25s+1)(.01s+1))$ is chosen.

Design Equation

The design equation $G = A^{-1}P^{-1}Q$ defines controller dynamics where $Q = T(I - HST)^{-1}$. Using the above T , A , S and P^{-1} matrices and choosing $H = I$, the controller $G(s)$, simplified by cancellations and approximations, turns out to be

$$G \approx \begin{bmatrix} -1.2(.11)(.05) & 3.2(.12)(.05) & 152(.05) & 10(.02)(.05)(.5)/(.16) \\ -.001(.66)(.2) & .002(-.17)(.2) & -.46(.1)(.2) & .004(.01)(.2)(.5)/(.16) \\ -.16(.24)(.1) & .16(-.01)(.1) & -16(.1) & .21(.01)(.1)(.5)/(.16) \\ -.013(.06)(.1) & -.13(.40)(.1) & -.49(-.06)(.1) & .19(.02)(.1)(.5)/(.16) \end{bmatrix}$$

$s (.01)$

Verification

System output response of the F100 engine using the above controller was verified by CSMP simulations. Command responses and decoupling for a step request of 4 PSI P_7 augmentor pressure and for a step request of 50 degrees FT1T temperature are shown in Figures 11 and 12 respectively.

SUMMARY REMARKS

Linear multivariable control synthesis with transfer functions appears to be feasible and practical. An output response synthesis method was described using two basic equations both featuring the inverse of the plant transfer function matrix.

The plant inverse matrix is key to multivariable transfer function synthesis. Its existence assures possibilities for plant trackability and decoupling; and, in factored form, it indicates plant zeros, cancellations and potential performance tradeoff to simplify the controller.

Transfer function synthesis builds on classical transfer function concepts, is easy to understand and contacts modern theory. Features include direct design of output response, cancellation and approximation and insight on response adjustments to simplify controller dynamics. The possibility to include both sensitivity specifications and response specifications looks promising and is under study.

Transfer function synthesis is applicable to gas turbine propulsion system design.

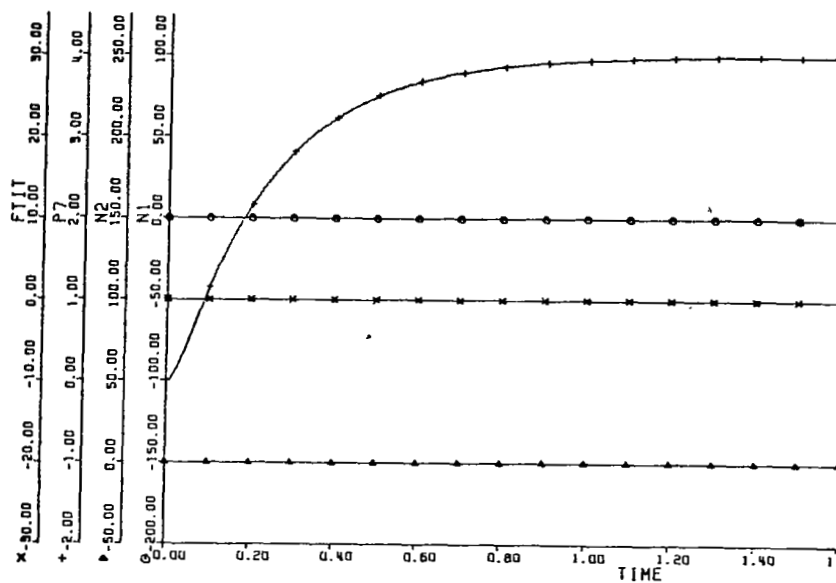


Figure 11. 4 PSI P7 Step

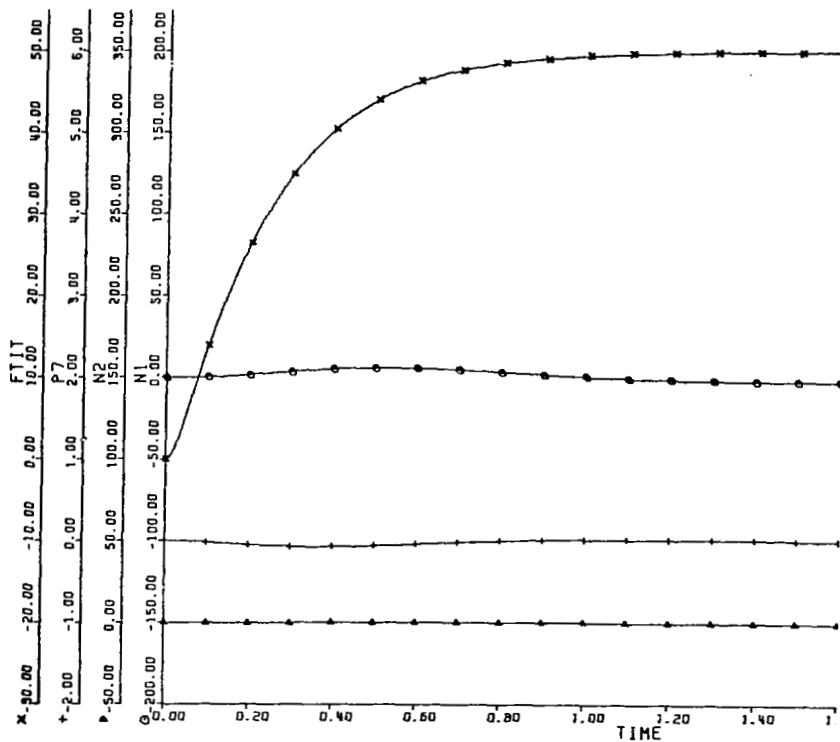


Figure 12. 50 Degree FTIT Step

REFERENCES

1. Boksenbom, A. A. and Hood, R., "General Algebraic Method Applied to Central Analysis of Complex Engine Types," NACA Technical Note 1908, National Advisory Committee for Aeronautics, Washington, D. C., July 1949.
2. Feder, M. S. and Hood, R., "Analysis for Control Application of Dynamic Characteristics of Turbojet Engine with Tail Pipe Burning," NACA Technical Note 2183, Washington, D. C., September 1950.
3. Truxal, J. G., Automatic Feedback Control System Synthesis, McGraw-Hill, New York, 1955.
4. Peczkowski, J. L. and Sain, M. K., "Linear Multivariable Synthesis with Transfer Functions", Proceedings, International Forum on Alternatives for Multivariable Control, National Engineering Consortium, pp. 111-127, October 1977.
5. Peczkowski, J. L., Sain, M. K., and Leake, R. J., "Multivariable Synthesis with Inverses," Proceedings, Joint Automatic Control Conference, June 1979, pp. 375-380.
6. Rekasius, Z. V., "Linear Multivariable Control - A Problem of Specification," Proceedings, International Forum on Alternatives for Multivariable Control, National Engineering Consortium, pp. 101-110, October 1979.
7. Wonham, W. M. and Morse, A. S., Decoupling and Pole Assignment in Linear Multivariable Systems: A Geometric Approach, "SIAM Journal of Control, Vol. 8, pp. 317-337, 1970.
8. Leake, R. J., Sain, M. K., and Peczkowski, J. L., "Step Trackable Linear Multivariable Plants", International Journal of Control, 1979, in Press.
9. Wyman, B. F. and Sain, M. K., "Essential Right Inverses and System Zeros", Proceedings 18th IEEE Conference on Decision and Control, pp. 23-28, December 1979.
10. Bengtsson, Gunnar, "Feedback Realizations in Linear Multivariable Systems," IEEE Transactions on Automatic Control, Vol. AC-22, No. 4, August 1977.
11. Miller, R. J. and Hackney, R. D., "F100 Multivariable Control System Models/Design Criteria," Tech. Rept. AFAPL-TR-76-74, November 1976.